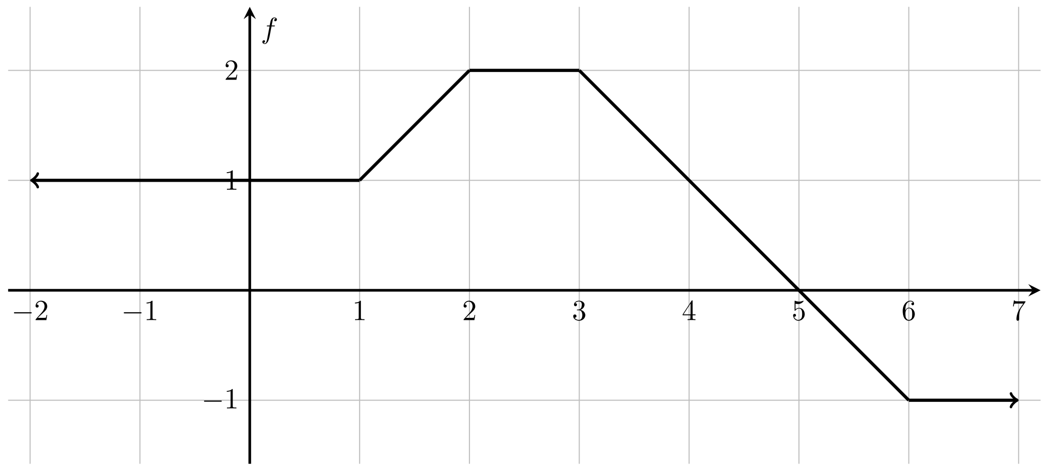
Area accumulation functions – an introduction

Given a function $f(x)$, we create a new function $F(x)$ by evaluating how much area is accumulated under $f(x)$.

1. Example:



(a) Define $F(x) = \int_0^x f(t)\,dt$. Evaluate the following:

$F(0) =$ $\qquad$ $F(2) =$

$F(1) =$ $\qquad$ $F(-1) =$

(b) Shade in and find the area represented by $F(3) - F(1)$.

(c) Find a formula for $F(x)$ between $x = 0$ and $x = 1$

(d) Give two values at which $F(x) = 0$. (Hint: assume the graph continues to the right.)

(e) Which is larger: $F(3)$ or $F(4)$? Explain.

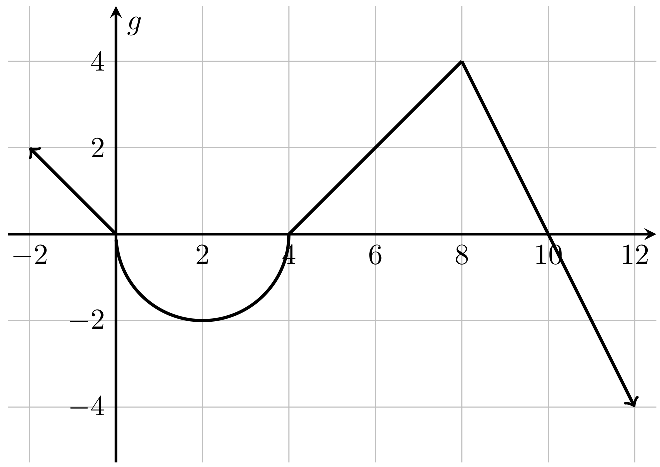
(f) Which is larger: $F(5)$ or $F(6)$? Explain.

(g) Give open intervals on which $F(x)$ is increasing. Explain.

(h) $F(x)$ has a local extremum at $x = 5$. Is it a maximum or a minimum? Explain.

(i) $F(x)$ is increasing at both $x = 1$ and $x = 2$. At which value is $F(x)$ increasing faster? Explain.

2. g is a piecewise function composed of line segments and a semi-circle.



(a) $G(x) = \int_4^x g(t)\,dt$

$G(4) =$

$G(10) =$

$G(12) =$

$G(0) =$

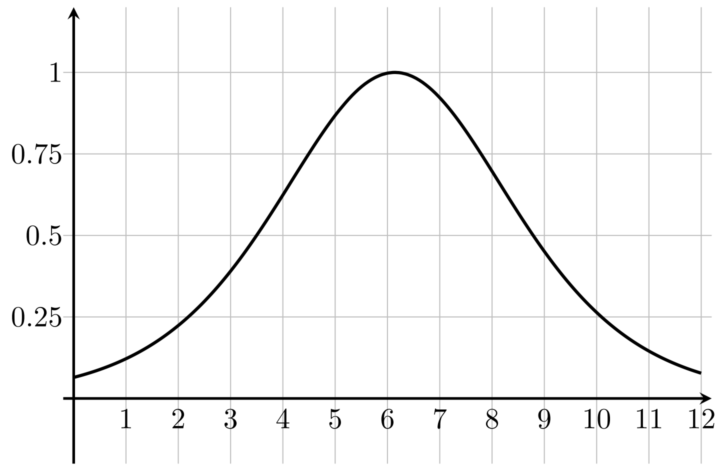
(b) On what open intervals is $G(x)$ increasing? decreasing?

(c) Find all local extreme values of $G(x)$ by determining where $G(x)$ switches from increasing to decreasing and from decreasing to increasing.

(d) What are the critical numbers of $G(x)$?

(e) On the interval from $[0, 12]$ where is $G(x)$ increasing fastest?

3. The peak of Boulder's epic rainstorm of 2013 occured between 4pm, Sept 12, and 4am, Sept 13. During those 12 hours the rate of rainfall can be modelled by $r(t) = \dfrac{240e^{2t/3}}{\left(60 + e^{2t/3}\right)^2}$ in inches per hour, where $t = 0$ represents 4pm on Sept 12.



Let $R(x) = \displaystyle\int_0^x \frac{240e^{2t/3}}{\left(60 + e^{2t/3}\right)^2}\, dt.$

(a) Use the graph to estimate $R(4)$. What does it represent? (include units)

(b) Use technology to calculate $R(12)$. What does it represent? (include units)

(c) What does $R(x)$ represent?

(d) Where is $R(x)$ changing the fastest?